USN | | | | | | | | | | 10MAT41

Fourth Semester B.E. Degree Examination, June/July 2014

Engineering Mathematics – IV

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions, selecting at least TWO questions from each part.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

PART – A

1 a. Obtain a solution upto the third approximation of y for x = 0.2 by Picard's method, given that $\frac{dy}{dx} + y = e^x$; y(0) = 1. **(06 Marks)**

b. Apply Runge-Kutta method of order 4, to find an approximate value of y for x = 0.2 in steps of 0.1, if $\frac{dy}{dx} = x + y^2$ given that y = 1 when x = 0. **(07 Marks)**

c. Using Adams-Bashforth formulae, determine y(0.4) given the differential equation $\frac{dy}{dx} = \frac{1}{2}xy$ and the data, y(0) = 1, y(0.1) = 1.0025, y(0.2) = 1.0101, y(0.3) = 1.0228. Apply the corrector formula twice. **(07 Marks)**

2 a. Apply Picard's method to find the second approximation to the values of 'y' and 'z' given that $\frac{dy}{dx} = z$, $\frac{dz}{dx} = x^3(y+z)$, given y = 1, z = $\frac{1}{2}$ when x = 0. **(06 Marks)**

b. Using Runge-Kutta method, solve $\frac{d^2y}{dx^2} - x\left(\frac{dy}{dx}\right)^2 + y^2 = 0$ for x = 0.2 correct to four decimal places. Initial conditions are x = 0, y = 1, y′ = 0. **(07 Marks)**

c. Obtain the solution of the equation $\frac{2d^2y}{dx^2} = 4x + \frac{dy}{dx}$ at the point x = 1.4 by applying Milne's method given that y(1) = 2, y(1.1) = 2.2156, y(1.2) = 2.4649. y(1.3) = 2.7514, y′(1) = 2, y′(1.1) = 2.3178, y′(1.2) = 2.6725 and y′(1.3) = 3.0657. **(07 Marks)**

3 a. Define an analytic function in a region R and show that f(z) is constant, if f(z) is an analytic function with constant modulus. **(06 Marks)**

b. Prove that $u = x^2 - y^2$ and $v = \frac{y}{x^2 + y^2}$ are harmonic functions of (x, y) but are not harmonic conjugate. **(07 Marks)**

c. Determine the analytic function f(z) = u + iv, if $u - v = \frac{\cos x + \sin x - e^{-y}}{2(\cos x - \cosh y)}$ and f(π/2) = 0. **(07 Marks)**

4 a. Find the images of the circles |z| = 1 and |z| = 2 under the conformal transformation $w = z + \frac{1}{z}$ and sketch the region. **(06 Marks)**

b. Find the bilinear transformation that transforms the points 0, i, ∞ onto the points 1, –i, –1 respectively. **(07 Marks)**

c. State and prove Cauchy's integral formula and hence generalized Cauchy's integral formula. **(07 Marks)**

PART – B

5 a. Obtain the solution of the equation $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+\left(x^2-\frac{1}{4}\right)y=0$. **(06 Marks)**

b. Obtain the series solution of Legendre's differential equation,

$$(1-x^2)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+n(n+1)y=0$$ **(07 Marks)**

c. State Rodrigue's formula for Legendre polynomials and obtain the expression for $P_4(x)$ from it. Verify the property of Legendre polynomials in respect of $P_4(x)$ and also find $\int_{-1}^{1}x^3P_4(x)dx$. **(07 Marks)**

6 a. Two fair dice are rolled. If the sum of the numbers obtained is 4, find the probability that the numbers obtained on both the dice are even. **(06 Marks)**

b. Given that $P(\overline{A}\cap\overline{B})=\frac{7}{12}$, $P(A\cap\overline{B})=\frac{1}{6}=P(\overline{A}\cap B)$. Prove that A and B are neither independent nor mutually disjoint. Also compute P(A/B) + P(B/A) and $P(\overline{A}/\overline{B})+P(\overline{B}/\overline{A})$. **(07 Marks)**

c. Three machines M_1, M_2 and M_3 produces identical items. Of their respective outputs 5%, 4% and 3% of items are faulty. On a certain day, M_1 has produced 25% of the total output, M_2 has produced 30% and M_3 the remainder. An item selected at random is found to be faulty. What are the chances that it was produced by the machine with the highest output? **(07 Marks)**

7 a. In a quiz contest of answering 'Yes' or 'No', what is the probability of guessing atleast 6 answers correctly out of 10 questions asked? Also find the probability of the same if there are 4 options for a correct answer. **(07 Marks)**

b. Define exponential distribution and obtain the mean and standard deviation of the exponential distribution. **(07 Marks)**

c. If X is a normal variate with mean 30 and standard deviation 5, find the probabilities that (i) $26\le X\le 40$, (ii) $X\ge 45$, (iii) $|X-30|>5$. [Give that $\phi(0.8)=0.2881$, $\phi(2.0)=0.4772$, $\phi(3.0)=0.4987$, $\phi(1.0)=0.3413$] **(06 Marks)**

8 a. Certain tubes manufactured by a company have mean life time of 800 hrs and standard deviation of 60 hrs. Find the probability that a random sample of 16 tubes taken from the group will have a mean life time (i) between 790 hrs and 810 hrs, (ii) less than 785 hrs, (iii) more than 820 hrs. [$\phi(0.67)=0.2486$, $\phi(1)=0.3413$, $\phi(1.33)=0.4082$]. **(06 Marks)**

b. A set of five similar coins is tossed 320 times and the result is:

No. of heads:	0	1	2	3	4	5
Frequency:	6	27	72	112	71	32

Test the hypothesis that the data follow a binomial distribution. [Given that $\psi^2_{0.05}(5)=11.07$] **(07 Marks)**

c. It is required to test whether the proportion of smokers among students is less than that among the lectures. Among 60 randomly picked students, 2 were smokers. Among 17 randomly picked lecturers, 5 were smokers. What would be your conclusion? **(07 Marks)**

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